

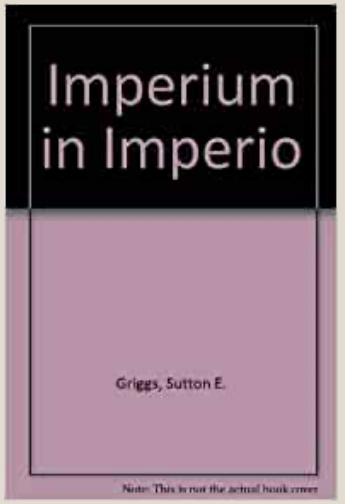
An overview of the paper

> The challenges of basing a full mathematics course entirely on realistic examples

about a two-year struggle with the development of a first-term course containing several new (or at least: rare) pedagogical elements.

Focus will be on the use made of results obtained in pedagogical research carried out prior to, as well as along with, the writing process and the actual teaching.

In particular, it will be argued that the work – which is quite substantial – could not have succeeded without these results and the experience gained in the search.




*Imperium in imperio*



# The course



The centrepiece is the triple of courses

62168 M Calculus (and some linear algebra)

62168 S Statistics

62169 Calculus in two variables + Differential Equations

taught to first-term student of Global Business Engineering at DTU Engineering Technology.

The aim of the course is to raise the students basic mathematical skills from Danish *Gymnasium* level B to level A, prepare them for the two mandatory courses 01901 and 01920 ***and*** provide them with an impression of the use of mathematics is real-life engineering.

The decision to base all exercises on 'realistic examples' was made from the outset (May 2020)

The word 'realistic' here means something like *tangible*

The two courses 62168M and 62168S are taught together during the 13-week term.

62169 is taught during the three-week term.

*Gymnasium* = upper secondary school.

The aims are described in the Course Base and are legally binding



# First challenge: The course structure



Prior to anything being written down, the content of the course-to-be was analysed top-down and broken into *Unit Content Objects*, nuggets just small enough to be written – but also in principle to be read and grasped – in one go. The course was next assembled bottom-up from these units.

Some modifications are necessary, but also possible, as this process gains momentum. The process is described in Hansen: *Unit Learning Object Analysis and Synthesis of an Elementary Course*, Proceedings ED-MEDIA 2003.

Each object occurs thrice: In a presentation (such as the present), in far more detail in the course notes, and as a training exercise. These three carry the same identifier in the form of three digits, such as 3.1.2

Content objects were once called Learning objects; but as the leárning takes place inside the learner's head they are now called content objects.

There is a – now obsolete – standard called SCORM for the definition and description of content objects. The use of the name is less restricted here

The author has analysed and synthesised some 20 courses this way



# Second challenge: Assembling the objects



There is considerable freedom in the writing of notes – and far less in the writing of presentations:

A section of the notes can in principle be as long as it needs to be in order to explain the content. In practice, it will range from 1 to 3 pages, rarely exceeding 3. To this must be added introductions.

On the other hand, presentations are subject to a (self-imposed) restriction to three PowerPoint slides per unit object in a layout similar to the present.

The author once reproduced the entire IPMA project management skill specification as 150 21-line 'minute speeches' (Kahn-Panni) and thus has practiced writing within narrow limits. Other than the resulting notes, no attempt has been made to document this particular effort...

**Good** templates are the productivity boosters *par excellence*

To some extent, once the analysis has been carried out, each unit of the notes, presentations or exercises can be written independently of the rest, i.e. the order is immaterial.

In practice, material was written in the order it was needed, as development took place while the course was being taught



# The third challenge: Teaching with models



Ideally, the notes themselves should be based on models – realistic models – only. However, the author has taught mathematics for 46 years and firmly believes that:

- The exposition must be "purely theoretical"
- The understanding comes with the subsequent study of examples of gradually increasing degree of difficulty *based on the realistic examples*
- The student must read the exposition again after having studied the examples – and possibly reiterate this process

In practice, many student fail to read the exposition (or to attend the lecture, which is a 'live exposition', or even both). There is no known cure for absenteeism.

There is no balm in Gilead, either

Basing *the notes* on realistic examples only would be a much larger effort.

In fact, it is not clear whether this could be done at all, seeing that mathematics ***is*** abstraction.

And just in case you haven't come across it, the negro spiritual is:

There is balm in Gilead,

To make the wounded whole;

There's power enough in heaven,

To cure a sin-sick soul.



# The fourth challenge: Teaching with unit objects

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1st challenge
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Conversion
ROME
6th challenge
The exams
Opgave
Noter
Emnevalg
Værktøj
Lyt
Test
Elefant

Each of the two parts of 62168 contains 108 unit content objects. (A similar course taught to a *Gymnasium* class contained more than 150 such units).

The synthesis of the course adheres strictly to the Vygotskian idea of a *Zone of Proximal Development*, trying to add thin layer by thin layer to the student's understanding of the subject while making use of the effect of repetition-with-small-variations:

The course has three part: easy – less easy – approaching the level of the later courses.

Within each part are the same three subjects: expressions, differentiation and integration

Each of the are split in four, presented as short lectures; and within each lecture, three unit objects are explained.

The students are told from the outset that there is 'enough material' and that they must make their own choices.

What is stated here hold for 62168 M, the first mathematics course.

For 62168 S, the statistics course, things are only sightly different; and for 62169, one of the three parts is pure repetition of 62168 M.

The students are thus exposed to the core issues four times.

And each unit is explained in presentation, notes and an exercise, remember



# A note on e-learning



The very solid structure of the course serves another purpose, not visible to the uninitiated:

The presentation is available in a digital form; the notes are actually written directly in the CAS tool employed, as are the exercises, i.e., they are eminently interactive; and the overview is a table (in MS ® Word ®, but it could be written in many other ways).

The only thing needed to convert the material into a viable e-learning 'platform' is that the elements be tied together by the addition of hyperlinks that will allow the student to switch between the representations of each unit, as has been done (successfully) on a small scale with material taken from other courses.

A presentation of this aspect will be given in the SAI Computing Conference, June 22-23, London.

The teacher-author must keep this in mind, the student needn't know

In the statistics course, the students can choose to work with the exercises in a spreadsheet, rather than in the CAS tool.

This makes no difference to the statements made here.

Also, presentations can be doubled or "shadowed" by a version with built-in "speak". This has proved popular with students but is time-consuming in the creation phase



# The fifth challenge: Conversion to e-learning



If you want to convert a course material built over a structure like the one described and with the full use of construction from unit content objects, all you have to do is to connect everything by adding hyperlinks; ***but***:

To many users the result will perhaps seem like little more than "electrified paper".

The addition of a voice-enhanced parallel presentation has been welcomed, but few users seem to appreciate the magnitude of the effort to do so.

If you are skilled with some media other than written text and static graphics, by all means add them, but maintain the structure.

If you plan to do this on a large scale, be careful that you get the time allocation right...

Of the 12-15 courses developed within the structure described here, 3 or 4 have had one or more of the e-learning features added – and/or have had links to interactive tests or other niceties.

The warning is the same: Plan this as a option from the beginning and make sure the structure allows it; and as far as possible, keep it an optional extra that can be added little at a time.

This is documented in the SAI Computer Conference paper, see above



# The **ROME** project



The **ROME** project – **R**epository **O**f **M**odel **E**quations – is a one-man endeavour to unearth and classify "as many as possible" model equations from the engineering literature – with some natural science, social science and financial studies thrown in.

The equations by their very nature describe 'realistic settings' and are therefore suited as starting points for the construction of exercises for use in courses like the ones described here. The only requirement is that they be "teachable", i.e., that their context can be explained in short, quickly understood terms and that their analysis is within reach of someone having grasped the concepts and methods of the course up to the ponit where the equation is introduced.

This can be done – but there is no systematic approach...

And yes, the **ROME** project will be documented "in due course"

The author recommends that one keeps *models* and *modelling* apart.

Mathematical modelling is a discipline of its own – and very difficult, as it requires mathematical maturity and a solid grounding in the discipline to which the model belongs.

**Please note**: Interested readers should contact the author at psha@dtu.dk



# The sixth challenge: Writing the exercises



Writing the exercises and finding the subjects – or finding the subjects and writing the exercises – can hardly be separated as challenges:

Not until you are right in the middle of one of these do you realize that you have – or sorely miss – a particular kind of exercise, representing a particular content object as a challenge at the proper level of difficulty.

As already noted, there is no systematic way of handling this, not even of searching for a missing piece of the puzzle. The author, who at the time of writing this has spent five years on finding material for 9-10 courses, now has a collection large enough to scare the faint of heart.

As for the writing itself, it has proved remarkably easy – the students will argue that the reading is a nightmare...

For as long as the author has taught mathematics, students have clamoured for "practical examples".

And now, when they are given practical examples by the bushel, they long for x's and y's.

The pedagogical literature is likewise divided; and there seems to be no clear winner.

In the evaluation, one student wrote: "And there were hardly any practical examples" (-?!-)



# The exams



In keeping with the decision to let the students make as many choices as possible regarding the material with which they prefer to work and how the go about it, the exams are as follows:

62168 S: Each student team writes three reports on subjects they have chosen from an ever-growing collection of report assignments written by the author. On the exam day, they draw a number 1-3 and present their report with that number

62168 M: Students hand in around 30 assignments during the term. Each student draws a number 1-30 and presents his or her solution to that assignment

62169: Written exam. The set consists of 24 problems in 8 subject groups and 3 levels of difficulty. The students solve "as many as they can", choosing freely

The latter exam type was documented in a 2003 report, see pre-print

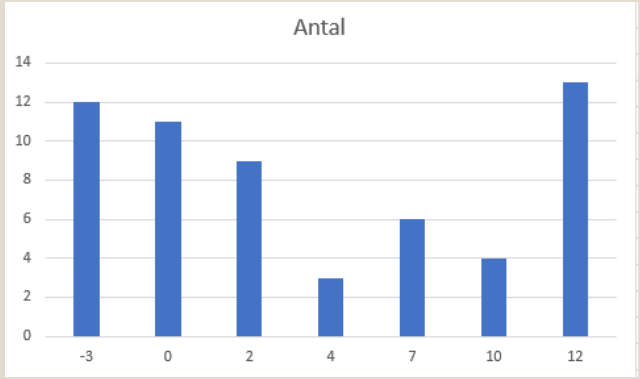
This combination of exams tend to give strong students a chance to display their talents and weaker student to pass by simple diligence.

Student who have displayed a marked lack of interest tend to fail – and this can probably not be remedied.

Altogether 1600+ pages, 120 PowerPoints, etc.

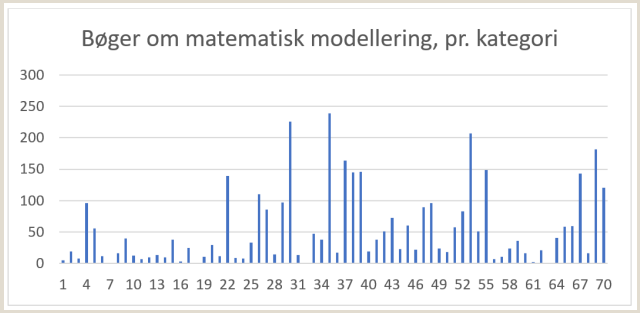


1. Strong students pass with high marks, weak but diligent students pass with acceptable marks – and the results of strong, diligent students transgress all expectations




January 2021 marks after the exam of 62169

2. (About **ROME**) It turns out that textbooks on different engineering disciplines place markedly different emphases on the use of models




Textbooks on (mathematical) modelling, by engineering discipline

3. The course no longer exists: Students without a level A mathematics exam are no longer admitted to DTU. The material presented here is therefore no longer in use.

But it was fun while it lasted...

Finis


## References

- See the pre-rpint

Elefant

Fra Internettet